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Policy selection by structural mapping

By B. H. P. Rivett
University of Sussex

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The paper is concerned with the selection of 'best' policies from a range of alternative possibilities where each policy is assessed in terms of its degrees of attainment of a set of objectives. The method proposed is based on multi-dimensional scaling techniques as further developed by Kendall for drawing maps based on fragmentary information. In the application discussed in this paper the input information is solely a list of those pairs of policies which have consequences which are approximately equally attractive. It is shown that such information can be sufficient to draw maps of the policies in such a way that two small groups of policies appear consistently at the extremes of the maps. It is suggested that these groups should normally correspond to the 'best' and 'worst' policies. In other words, an axis of preference might be deduced solely from statements of indifference and a comparison with the utilities of the policies appears to give credence to this.

1. Introduction

A topic of contemporary concern is that of how to deal with decision making problems when more than one objective is involved. In this paper we shall define an *objective* as a desirable quality of a system, which is preferably measurable in a particular unit of measure. A *policy* will be a set of decisions which could be applied to the system and a *degree of attainment* of an objective is the (preferably measured) level achieved by a policy applied to the system. In general, therefore, a policy could be assessed in terms of the degrees of its attainment of a set of objectives. In this paper we shall (apart from the cost-benefit analysis critique) assume that we have taken every possible opportunity of subsuming any given objective into another – that is every possible conversion of the different units of measure has taken place and we are left with a residuum of incommensurable objectives. At this point there is a paradox. For if we have a truly incommensurable set of objectives, then how can we choose between different policies which achieve them to different extents? Moreover is it not *by definition* impossible to create any other metric which apparently measures incommensurables? And yet choice has to be made. It is for this reason that what has been named 'satisficing' becomes so important – the generation of policies which simply satisfy constraints on the desired degrees of attainment of the objectives. The sole reason, perhaps, for persisting in attempting to optimize in the face of this logical objection is that mathematical methods are stronger at selecting independent variables that give extreme values of dependent

variables than they are at generating independent variables for which the dependent variables fall within given ranges. Even in this 'satisficing case', however, there is an attraction in any method which might reduce the number of alternative policies which have to be considered and one advantage of the method suggested in this paper is that it might not only do that but also that it suggests a very limited subset of these alternative policies from which the 'best' should be chosen or between which the 'best' will strike a (subjectively assessed) balance.

2. Methodological comparisons

In surveying the alternative approaches to the choice problem, the common feature they seem to share is a reluctance on the part of decision makers actually to use them. This may be because decision makers are obtuse, and fail to observe the gold in our offerings. It may, however, be more likely that what the decision maker is asked to assume in order to use these methods strains his credulity.

The following section examines briefly but critically three alternative methods of optimizing in multi-criteria situations – the use of utility, cost benefit and linear programming. We shall then compare the assumptions of these methods and formulate a possible alternative.

(a) *Utility*

There are three problems arising with the use of utility. First, the axiomatic demands (which are listed in von Neumann & Morgenstern 1964), are well known, but impose severe behavioural constraints (see, for example, Rivett 1972). Secondly, the creation of the utility of a decision taker by offering him a series of hypothetical gambles may seem unreal to a man not used to thinking in probability terms. Finally, since documented evidence of case studies using utility in any significant situation is, to say the least, rare, there is an air of 'playing games with reality' about the process.

The main point is not to declare utility as anathema; this would be silly. It is to comment that its central requirement is quite severe; we have to be able to rank alternatives, and then to measure the 'distance' between alternatives in a subjective way by hypothetical gambles.

(b) *Cost benefit analysis*

Cost benefit analysis forces the analyst and decision maker to ignore the incommensurable nature of the variables by requiring them to use money as a transfer unit. It is not the purpose of this paper to repeat the arguments regarding this technique but to observe that the requirement it makes is very strong. Namely, that we can use money as a transfer function linking all the objectives – including objectives which are cultural or artistic (for example in the third London airport

inquiry, the use of money as a measure of the amenity loss incurred by pulling down a Saxon Church). If these strong assumptions are acceptable then the problem of choice becomes one involving only a single unit of measure.

(c) *Linear programming*

There are a number of alternative methods, all making the same sort of demand. A typical example would place each policy, assessed by n criteria, as a point in an n-dimensional space. The set of all policies would be reduced to the efficient set and the analyst then takes the decision maker on a tour of the vertices, proceeding from one to the next by some form of steepest ascent method. The requirement here is less than the other two, but it still involves ranking and choosing between groups of alternative policies in moving from one vertex to the next (see, for example, the work of Zionts & Wallensius 1976).

At first sight it seems that a problem such as this, involving for each policy, a quantitative statement of its degrees of attainment of (say) n different objectives, could be approached by principal component analysis or canonical analysis. However, at the very least such approaches would involve these degrees of attainment being additive, which means they would be expressed as utilities and so would require the stronger assumptions of utility theory. Additionally it must be remembered that the points in such analyses each represent the consequences of a particular policy. Since these policies are selected *a priori* by the decision maker and may be added to at his whim, it is difficult to see what meaning would be attributed to the components or factors when they were derived.

All this is not to decry these other approaches, and clearly comparative research would be important. But this lies outside the scope of our research in which the first question is what is the weakest requirement we can make in order to use a method of policy selection? One weak requirement would be merely to inquire for any particular policy, what other policies give outcomes that are approximately equally as attractive. This does *not* mean outcomes that are similar, for two policies may have quite different outcomes which when taken as packages are equally attractive.

3. The decision process

We have referred to the decision choice problem caused by variables being incommensurable and have observed the constraints imposed on the decision-taker by some of the existing methods. These methods require strong assumptions and we can place assumptions in descending order of strength.

(*a*) The consequences of decisions can all be expressed in terms of one variable (e.g. money) without time or probability entering into the matter.

(*b*) as in (*a*), but with time and/or probability now playing a rôle.

(*c*) The consequences of decisions are heterogeneous and incommensurable, but they can be placed in a rank order, and a metric can be devised to show how 'close' in *attractiveness* any one decision is to another (e.g. by utility theory).

(*d*) As in (*c*), but without the rank ordering constraint, and in its place merely the hypothesis that some decisions have consequences which taken as a whole, by and large, might be described by some geometric configuration where *proximity* in the geometry will be associated with *near indifference* from the standpoint of the decision-taker. This paper will attempt to suggest an approach to (*d*).

4. The recovery of structure from fragmentary information

The key references are to the work of Shepard and Kruskal in developing the technique of multi dimensional scaling (MDSCAL). This technique is outlined in Kruskal (1964*a*, *b*), which in its turn lists other basic papers. MDSCAL has been utilized in the field of history and archaeology (Hodson, Kendall & Tautu 1971) and among the applications is that of deducing a map from fragmentary information. The present method, devised by Kendall, makes use of the MDSCAL algorithm and takes as input information *only* the ranked order of 'pseudo-distances' between all pairs of a set of n points. Starting from a random initial configuration, a series of iterations deduces an approximation for the location of these n points in a K dimensional space. There are different modes of use of the algorithm; Kendall in Hodson *et al.* (1971) suggests that if a K-dimensional map is required, an initial fit should be made in $(K+1)$ dimensions. This is then projected parallel to the weakest of the $(K+1)$ principal components and a further fit then made starting with this initial projection.

As a side effect of his work in archaeology and history, Kendall (1971, 1975, 1976*a*, *b*, 1977) has developed this in a number of important ways involving different concepts of 'pseudo-distance' to be used in the rank ordering of 'pseudo-distances' between pairs. Kendall (1971) shows how intermarriage rates between pairs of parishes in rural England in the period from about 1600 to 1850 can be standardized and co-ranked as a surrogate for a 'pseudo-distance' ranking to produce a surprisingly accurate map of the location of the parishes. A further example, using another concept of 'pseudo-distance', which we shall use again in this paper, is his use of the same algorithm to produce a map of the United States. In this we take as input data a listing of which pairs of the 48 states of the U.S.A. *abut* (omitting Alaska and Hawaii). The simplest definition of 'pseudo-distance' is a 'distance' of 1 between pairs of states which abut and a 'distance' of 2 for those which do not abut, but which are adjacent to a common adjacent state. Such a rank ordering of 'pseudo-distances' as this gives a large number of ties, but notwithstanding the very weak information, a remarkably accurate map of the U.S.A. is produced (see figure 1). A difficult problem in this work is that many 'pseudo-distances' come out to be equal. The severe effect of these ties has been controlled by Kendall's 'tertiary treatment of ties', for which see Kendall (1977). A brief description of this device is given in Kendall's Appendix to the present paper.

Kendall's program PREMAP receives as an input solely the immediate adjacen-

cies, and derives from these the second order 'adjacent adjacencies'. Starting from a random planar configuration of states, a series of iterations then draws together territories which are close in respect of all utilized proximity criteria. In all such mappings the final picture drawn will be one giving not only relative position but also a somewhat distorted relative distance. It will rarely be a perfect fit to the data but there are criteria by which the goodness of fit may be assessed (the 'stress' measure) (see Shepard, Romney & Nerlove 1972).

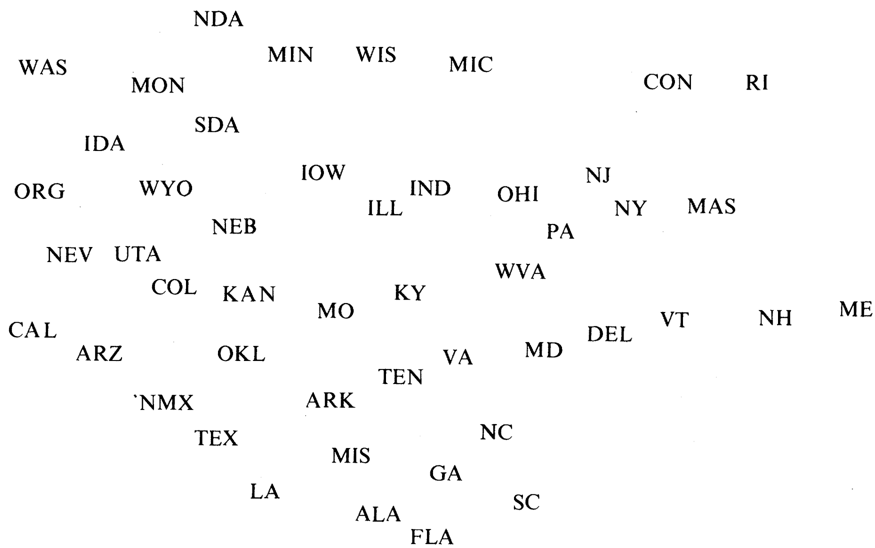

FIGURE 1. Globally processed map of the U.S.A. based on 'local' information (from Kendall 1977).

A study of figure 1 shows that the New England states have here been inverted with respect to the rest of the U.S.A. This is because New York State is a focal state through which all links between New England and the other states proceed, and hence we have effectively drawn two separate maps. It is then subject to chance whether New England comes out the same way up as the rest of the U.S.A., or (as here) not. Apart from this the only obvious defect in Kendall's map is the displacement northwards of New Jersey.

5. A POLICY SELECTION EXPERIMENT

There is an obvious possibility of applying the Kendall mapping procedures to policy selection. The striking point about that method is its successful mapping based on very weak information. Can we, therefore, state solely for each separate policy or a group of policies those other policies which have consequences which are by and large roughly similar in attractiveness (corresponding to the abuttal

information for the U.S.A.) and from this deduce a map? And if we can, what characteristics might we expect the map to possess?

It is important to remember that we are not linking together policies whose results are similar in their consequences. If we did then some form of multivariate or cluster analysis would be appropriate. We are solely linking them together when their consequences are similar in attractiveness. *Two policies can be alike in attractiveness, while quite different in their consequences.* This fact is absolutely central to the new approach to policy selection put forward here.

TABLE 1. HYPOTHETICAL TOWN OF BROVE: LEVELS OF ACHIEVEMENT OF OBJECTIVES BY ALTERNATIVE POLICIES

	coronary deaths within				
policy	one week, m	three years, n	infant mortality, p	old people's housing, N	defective infants, M
1	100	200	100	0	20
2	100	200	100	1000	23
3	100	200	70	0	32
4	100	200	70	1000	35
5	100	200	20	0	30
6	100	200	20	1000	40
7	300	100	100	0	15
8	300	100	100	1000	19
9	300	100	70	0	18
10	300	100	70	1000	20
11	300	100	20	0	22
12	300	100	20	1000	28
13	250	150	100	0	15
14	250	150	100	1000	20
15	250	150	70	0	21
16	250	150	70	1000	22
17	250	150	20	0	23
18	250	150	20	1000	29
19	100	100	100	0	22
20	100	100	100	1000	28
21	100	100	70	0	23
22	100	100	70	1000	30
23	100	100	20	0	38
24	100	100	20	1000	40

6. THE EXPERIMENT

It seemed appropriate therefore to formulate an example to test the method. We proposed the following for a *hypothetical* town of Brove. This town has a population of half a million and the authorities are considering spending £1.5M per year over the next few years. The money has to be spent to assist the population in health or housing and there are four main areas of concern.

(1) Coronary heart attacks. Such is the stress of living in Brove that there are currently 1000 people who suffer such attacks per year. Of these 300 die within a week and 200 die within the following 3 years.

(2) Infant mortality. Currently there are 10000 births each year and of these 100 infants die within the first 3 months (this includes the stillborn).

(3) Defective infants. In addition to these 10000 births, 50 children are born either spastic or with physical and mental defects due to lack of prenatal care.

(4) Old people's housing. At present it is estimated that in Brove there are 5000 people aged more than 70 who are in sub-standard housing. Many of these make

TABLE 2. POLICY INDIFFERENCE TABLE

1	2	5	7	13			
2	1	5					
3	4	6	7	13			
4	3	7	8				
5	1	2					
6	3	7	8	14	16		
7	1	3	4	6	9	17	
8	4	6	12	16			
9	7	11	13	22	23		
10	12	18	23				
11	9	19	22				
12	8	10	13	14			
13	1	3	9	12	17	18	20
14	6	12	18				
15	18	22					
16	6	8	18	20	22		
17	7	13					
18	10	13	14	15	16	21	
19	11	24					
20	13	16	22				
21	18	24					
22	9	11	15	16	20		
23	9	10					
24	19	21					

Interpretation: The above table indicates, for example, that the consequences of policy 1 are of the same attractiveness as the consequences of policies 2, 5, 7 and 13.

demands on the health services due to their housing conditions and their lack of ability to look after themselves. Hence it has been discussed whether special housing with resident nurses and wardens should be built for old people.

The different policies being considered will all make different inroads on the above four areas of concern. We define these inroads as follows:

(1) Coronary heart attacks. Number dying within one week, m, and number dying within 3 years, n.

(2) Infant mortality. Number of children dying within 3 months, p.

(3) Defective infants. Number of defective children born a year, M.

(4) Old people's housing. Number of old people accommodated in special housing, N.

Note that we would hope to reduce m, n, p and M, and to increase N.

A set of 24 alternative policies is proposed in each of which the total expenditure would be £1.5M. (This was obtained by assuming cost relations for various

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improvements from the base position in each of the five categories.) This set of policies is shown in table 1.

The author then took some time to consider the policies and stated in his own judgment which policies would have consequences of similar attractiveness. These are shown in table 2.

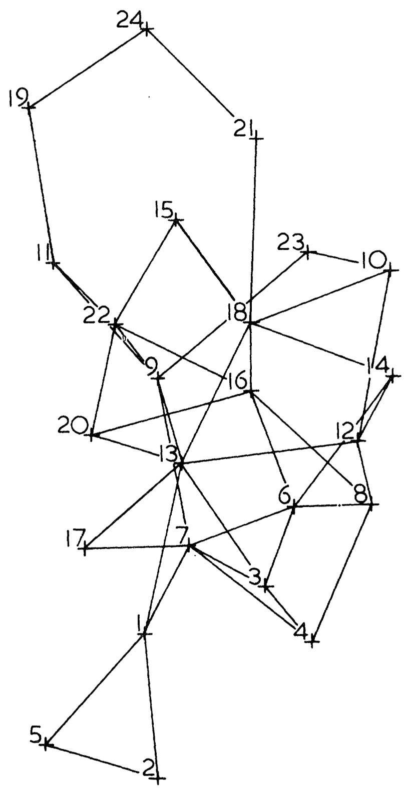

FIGURE 2. Brove: two dimensional policy map.
(M = 3, CTR = −1, run DGK(K189).

The programmes referred to in §4 were then applied with a set of seven different random starting conditions. A series of iterations produced maps of which figure 2 is typical (there was a strong similarity between all the final maps and this particular map). A line joining a pair of points corresponds to the author's 'indifference' as indicated in table 2. Note that the presence or absence of such line-segments corresponds exactly to the raw data on which alone the map itself is based. Thus the data are visible in full when the map is viewed (see table 2). These maps show that in all of the seven cases the extreme policies included (1, 2, 5) and (19,24). In all the maps, policies 19 and 24 were at one extreme end of the mapping (with 21 close by) and policies 1, 2 and 5 at the other end (with 4 close by). There is some suggestion therefore that the deduction from the author's 'indifferences'

is that 19 and 24 (and 21) lie at one of his preferences with 1, 2 and 5 (and 4) at the other end. The selection problem is therefore very much reduced. In all these computer runs we started with an initial random configuration in two dimensions and carried out a series of iterations in the same plane. As an additional

TABLE 3

	extreme points	
map 1 (figure 2)	1, 2 or 5	19 or 24
2	1, 2, 4 or 5	19, 21 or 24
3	1, 2 or 5	19 or 24
4	1, 2 or 5	19 or 24
5	1, 2 or 5	19, 21 or 24
6	1, 2, 4 or 5	19 or 24
7	1, 2, 4 or 5	19, 21 or 24

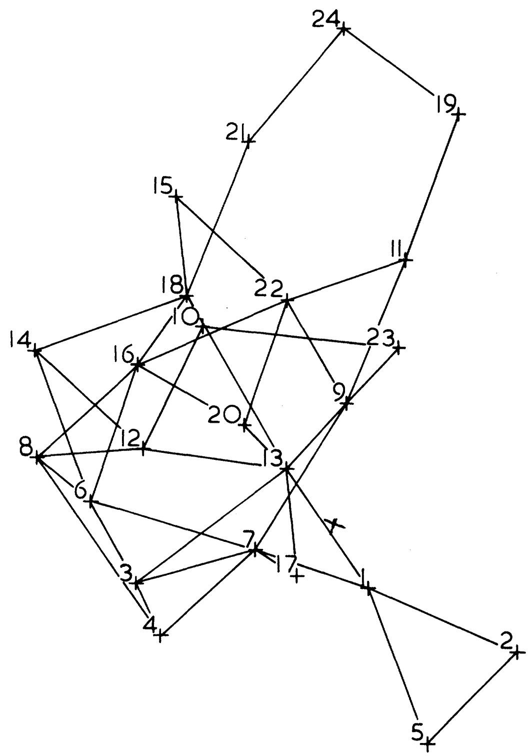

FIGURE 3. Brove: theee dimensional policy map projected on plane of first two components (M = 3, CTR = −1, plane (1, 2), run DGK (A066)).

test we also carried out a series of three dimensional fits, mapping by viewing the configuration along the three principal components. These results are shown in figures 3, 4 and 5. It is interesting to observe that figures 3 and 4 correspond well, particularly in the extremes, with the two dimensional map. The third figure is the

projection parallel to the strongest of the three components and hence is emphasizing a lower order of preferences. The most highly linked policy, 13, is centrally placed in all the maps. The remaining policies on the whole behave differently in figures 2, 3 and 4 on the one hand, and in figure 5 on the other. In figures 2, 3 and 4 the highly linked policies tend to go to the 'middle' of the map, and the less linked policies to the extremes, while in figure 5 this situation tends to some extent to be reversed.

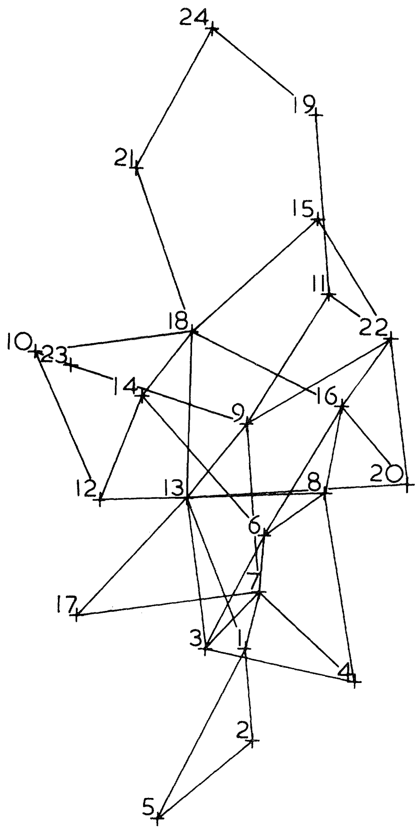

FIGURE 4. Brove: three dimensional policy map projected in plane of first and third components (M = 3, CTR = −1, plane (1, 3), run DGK (C071)).

To see this, look at policies 6, 7, 9, 16, 18 and 22 (which after policy 13 are the most strongly linked), and contrast their behaviour with that of 2, 5, 15, 17, 19, 21, 23 and 24 (which have just two links each). These considerations suggest a tendency for the 2-dimensional map to throw the less well-linked policies to the extremes, and when there are (as here) *two* extremes, they are segregated into two distinct groups, the identification of which is perhaps the principal fact to be learned from the maps.

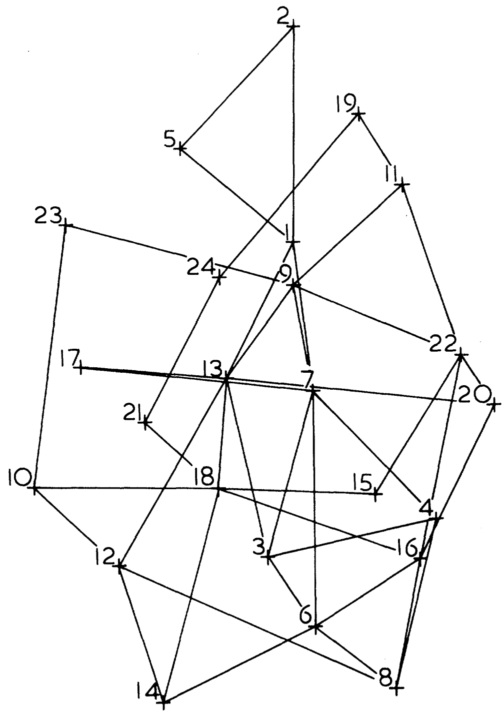

FIGURE 5. Brove: three dimensional policy map projected in plane of second and third components (M = 3, CTR = −1, plane (2, 3), run DGK (B069)).

7. THE UTILITY SOLUTION

After the author had stated his 'indifferences' in table 2, he allowed a four week gap to elapse (and worked on other things) so that his memory might forget any prejudices emerging from this study of similarities. He then formulated his utilities for the five different policy attributes and assigned utilities for the 24 policies, by adding these five utilities as shown in table 4. It is interesting to compare the policy maps with table 4, as in the nine maps (including figures 3 and 4) the extreme points were already defined and well differentiated from the other points.

From the utility table 4, it can be seen that policy 24 is clearly the best choice on utility considerations, followed by 21, 22 and 23. Policy 19, which is omnipresent above, merely has a slightly better than 'median' summed utility. (The median group actually consists of policies 10 and 17.)

At the other extreme of the map, policy 2 is of quite low utility, but on the other hand policy 4 is only on the low side, and 5 is *better* than median. The lowest on the utility scale appears on the maps near the low end of the 'middle', and is by no means an extreme point. Also policy 5 has a higher utility than policy 19.

Taking projections of the other 22 policy points on the line joining policies 2 and 24, and correlating the distance along the line of these points with their corresponding utilities gave a correlation coefficient of 0.46, significant at the 5% level.

There is clearly a relation between utility results and the mapping approach but it is evidently not a clearcut one, and it would indeed be surprising if they gave identical answers.

TABLE 4. PERSONAL ATTRIBUTE UTILITIES

	policy	$u(m)$	$u(n)$	$u(p)$	$u(N)$	$u(M)$	Σu	
L	1	1.85	1.00	1.00	1.00	1.75	6.60	
	2	1.85	1.00	1.00	1.20	1.68	6.73	
	3	1.85	1.00	1.47	1.00	1.45	6.77	
	4	1.85	1.00	1.47	1.20	1.38	6.90	
	5	1.85	1.00	1.95	1.00	1.50	7.30	
	6	1.85	1.00	1.95	1.20	1.25	7.25	
L	7	1.00	1.65	1.00	1.00	1.87	6.52	lowest
	8	1.00	1.65	1.00	1.20	1.78	6.63	
	9	1.00	1.65	1.47	1.00	1.80	6.92	
	10	1.00	1.65	1.47	1.20	1.75	7.07	
	11	1.00	1.65	1.95	1.00	1.70	7.30	
	12	1.00	1.65	1.95	1.20	1.55	7.35	
(L)	13	1.20	1.33	1.00	1.00	1.87	6.40	
L	14	1.20	1.33	1.00	1.20	1.75	6.48	
	15	1.20	1.33	1.47	1.00	1.72	6.72	
	16	1.20	1.33	1.47	1.20	1.70	6.90	
	17	1.20	1.33	1.95	1.00	1.68	7.16	
	18	1.20	1.33	1.95	1.20	1.52	7.20	
	19	1.85	1.65	1.00	1.00	1.70	7.20	
	20	1.85	1.65	1.00	1.20	1.55	7.25	
H	21	1.85	1.65	1.47	1.00	1.68	7.65	
H	22	1.85	1.65	1.47	1.20	1.50	7.67	highest
H	23	1.85	1.65	1.95	1.00	1.30	7.75	
(H)	24	1.85	1.65	1.95	1.20	1.25	7.90	

8. A SUGGESTED ROUTINE FOR PRODUCING MAPS

At present the method seems most applicable if one is considering choosing between say 15 and 40 alternative policies. If the number of alternatives is less than 15, it might be possible to fabricate more policies so as to enable a map to be drawn. If the number is more than 40, one might deal with a sub-set of these policies.

Suppose we number our policies 1, 2, ... For each policy in turn we list which of the other policies have results which, by and large, are equally attractive. For example for policy 1 we might have

1–3, 5, 7 meaning that when we, so to speak, hold policy 1 in our hands, policies 3, 5, 7 look equally attractive.

We do the same for each of the other policies in turn without reference to our previous selections (this is important). This might give us:

1–3, 5, 7
2–4, 7, 8
3–4, 5, 8, 10, etc.

We check this table for inconsistency. For example in the first row we have the link $1 \rightarrow 3$, but in row 3 we do *not* have $3 \rightarrow 1$. We separate out the consistent links from the inconsistent and test statistically whether our consistent links are merely random coincidences. If we survive this test we look at the inconsistencies each in turn and decide whether they do or do not show an indifferent pair, adjusting the table above accordingly. We now have a consistent indifference table corresponding to table 2 of this paper.

We now use the PREMAP program, which at present is available only in the Department of Pure Mathematics and Mathematical Statistics at Cambridge University, to derive the ranked order of pseudo-distances. This is used as an input to the MDSCAL program (Kruskal 1964*a*, *b*; Kendall 1976*b*), adapted to incorporate Kendall's tertiary treatment of ties, and a series of runs starting from different random configurations each yields a policy map. It appears that in general about 50 iterations will suffice for convergence.

One is, of course, concerned with the policies at the opposite ends of the map and if the final configurations show in a number of runs consistent groups at these extremes then it is suggested that the two groups be compared with each other to see which (if any) is the preferred group. It is possible however for a local optimum to be reached during an iteration and a point which 'should' be at an extreme can be held back from progressing towards it. This could be overcome by starting a further series of iterations with the point in question being transferred near to its 'correct' position. Naturally before taking such a drastic interference with the procedure one should have performed enough runs to feel confident where the 'correct' place is likely to be.

9. Discussion and conclusions

(i) Clearly at this stage, the application of the method proposed poses more questions than it answers. We need to know more about the robustness of the mapping solution but from these few runs the prospect of robustness in terms of the original configuration is encouraging. Indeed further work now being carried out, and to be reported subsequently, indicates that the removal of some of the alternative policies from the Brove example or the removal at random of up to 40 % of the links do not affect the configurations at the extremes, *providing* the number of links is not reduced below say three for each policy. When (as for Brove) this minimum is reduced to two the maps appear less robust but still with an approximate relation with the original map. At present however these results must be regarded as tentative.

(ii) In a few additional runs Kendall used a program due to E. M. Wilkinson which computes what is called the 'Wilkinson' metric. This is based on the totality of indirect links (not just the adjacencies and adjacent adjacencies). This when fed to MDSCAL gave broadly similar results.

(iii) The 3-dimensional runs indicated a 3-dimensional policy 'map' in which one

dimension (that stretching from policies 2 and 5 to policies 19, 21 and 24) accounts for by far the greater part of the variation, as it does in two dimensions, so that the conclusions do not appear to be a factitious consequence of confining the map to only two dimensions. (Incidentally all plots are shown with the principal axis 'vertical').

(iv) A question of interest is that of the number of dimensions in which the policy map should be drawn. If we are interested in extreme policies (e.g. but *not*, that is, the extraction of a small group of 'best' and 'worst') then a linear mapping should suffice. But by the nature of the mapping process, this line is contained in the plane in which the two dimensional map is drawn, as the principal component. In its turn the three dimensional maps include, as one of their three planes, the plane of the two dimensional map. Hence there is, at the least, no loss of information in proceeding to a higher dimension and in any practical case it might seem logical to start with a one dimensional plot and to increase the dimensions only so long as the decision maker(s) find the extra information helpful. The extra spatial feel gained in two dimensions appears helpful. Although arguments based on intuition are dangerous, the proximity criteria which would be the input to a single dimension map, 'should', one might feel, produce a mapping in which the 'most' preferred and 'least' preferred are towards opposite ends of the line. As this line is contained, as the principal component, in all the higher order maps, these maps should also show the same separation of 'best' and 'worst' along the direction of this same principal component (but see (v) below).

At this point, however, one should heed Kendall's warning (Kendall 1971, 1975) against the use of one dimensional maps; these are dangerous because they are *obliged* to reveal a linear arrangement whether this is implied by the data or not.

(v) In (iv) we tentatively suggested that the two extreme 'ends' of the policy map might prove always (as *here* confirmed by table 4) to represent the 'best' and the 'worst' groups of policies, respectively. However, we must be on our guard in case it should happen, in other less externally controlled situations, that there are two clearly defined extremes which are *alternative forms of 'worst'* (e.g. the 'extreme left' and 'extreme right'). In such a situation one might wish to form the decision-taking on the *median* region. (It could of course also happen that the two extremes are alternative forms of 'best', corresponding to two incompatible but individually attractive features.)

(vi) In terms of the derivation of table 2, the number of comparisons to be made will increase with the square of the number of policies. It was relatively easy (about 3 h work) to derive Table 2, and the computer runs only took a few seconds each in computation time. Consequently while a consideration of 50 policies (as in the mapping of U.S.A.) is practicable in computation time, the time involved in drawing up table 2 would increase by about fourfold, i.e. to something like 12 h. However, it would be rather silly to expect any method to carry out so complex a task as such a policy selection, without any work on the part of the decision analyst. In addition 'analyst fatigue' and its relation to prejudice would require serious study.

(vii) What is perhaps most encouraging about the method is the very weak demand it makes on the decision analyst and the decision maker. In fact it makes the weakest possible demand. The solution(s) stemming from the method cannot of course be regarded as optimal since we do not know in what respect optimality has been achieved. It will indeed be recalled that § 1 cast doubt on the concepts of optimality in multi-criteria choice. We might regard the solution(s) as indicating those policies (the 'median' group, or *one* of the 'extreme' groups) which are to be more closely investigated, on the basis of the internal logic of table 1, and, most importantly, on the concept of the 'distance' between two policies.

(viii) A final attractive feature of the method appears in the presentation of the results. Different table 2 type data from different decision takers will lead to different configurations. It might be that joint compromises would be more easily achieved from graphical plots. Alternatively, a compromise table 1 consisting of the common elements of different contiguities could lead to acceptable solutions.

The literature of decision analysis and management science has suffered so much from enthusiastic and uncritical advocacy of proposed solutions to this class of problem that one must be cautious in regarding yet another new proposal. We can say that this method is attractive, in that it recalls the appeal of early operational research to the methods of other sciences, and that it makes minimal demands on credulity. Any further claims must be subject to more detailed analysis and to the test of experience gained from real situations.

I should like to thank Professor D. G. Kendall, F.R.S., not only for his patience, advice and encouragement, but also for writing the necessary new programs and carrying out the computer runs. The variant of MDSCAL used in this investigation was that devised by Professor R. Sibson of the University of Bath, who kindly made it available to us. It accommodates the 'Tertiary treatment of ties' introduced by Kendall (1977) and used here (see Appendix) and includes elegant and useful modifications due to Sibson.

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Appendix. On the tertiary treatment of ties

By D. G. Kendall, F.R.S.

Professor Rivett has asked me to add a few technical remarks to his paper, especially in view of the fact that my article (Kendall 1977) will probably not appear before the present paper is published. Readers of Kruskal (1964*a*, *b*) will be familiar with the important distinction between the Primary and Secondary treatment of ties (PTT and STT). Very loosely speaking, the 'penalty' score which is to be minimized is increased (for STT) whenever two pairs of 'objects' having the *same* 'pseudo-distance' are represented by pairs of points in the map with *different* map-distances. In PTT, on the other hand, the constraints are much relaxed, and instead it is just required that, after all pairs of 'objects' have been assigned to 'tie-blocks' (object-pairs within a given tie-block having equal pseudo-distances), then the *average* point-pair-distance per tie-block is to increase in a manner which corresponds with the direction of increase of the pseudo-distances.

Now in the making of maps from pseudo-distances based on abuttals or adjacencies one normally starts off with pseudo-distances having just the two values, say

1 (implying 'adjacent')
and
∞ (implying 'non-adjacent', or 'not *known* to be adjacent).

In such a situation it is extremely helpful to use a hybrid between PTT and STT and so I have introduced what I call the *Tertiary* treatment of ties (TTT), in which, in the sample case just now alluded to, the machine manipulates the tie-block of adjacent pairs as if it were committed to STT and, the machine manipulates the tie-block of *non*-adjacent pairs as if it were committed to PTT. At my suggestion, Professor Robin Sibson very kindly made the programming adjustments necessary to permit the use of TTT (in a slightly more flexible form than that sketched here) when using the very attractive version of MDSCAL devised by him and now available at some computer installations in the U.K. (Universities of Bath and Cambridge).

In the work described by Professor Rivett there are 3 tie-blocks: 'adjacent', 'nearly adjacent', and 'not adjacent'. TTT is used here in such a way that STT applies to the first two of these, and PTT to the third.

In 10 further analyses conducted in the same way it was found that the arrangement of Professor Rivett's 24 policies along the main axis of variation was very